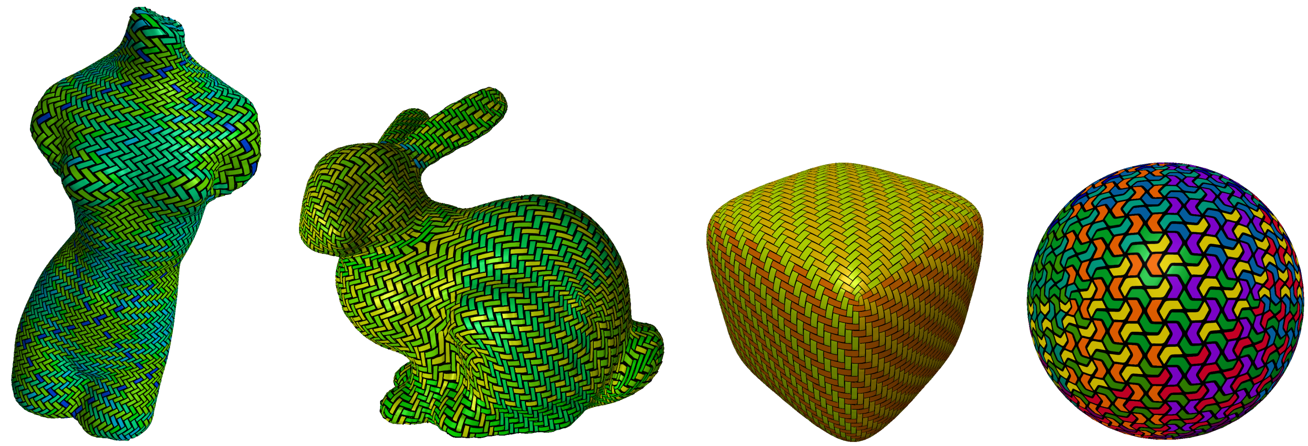
Cyclic Twill-Woven Objects

Ergun Akleman, Visualization Dept., Texas A&M University

Jianer Chen, Computer Science Dept., Texas A&M University

Yen-Lin Chen, Computer Science Dept., Texas A&M University

Qing Xing, Architecture Dept., Texas A&M University



(a) Venus (b) Bunny (c) Cubical (d) Geodesic Dome

Figure 1. *Examples of cyclic twill weaving on polygonal mesh surfaces constructed with the voting algorithm.*

Any arbitrary twist of the edges of an extended graph rotation system induces a cyclic weaving on the corresponding surface [Akleman et al. 2009]. This recent theoretical result allows us to study generalized versions of textile weaving structures as cyclic weaving structures on arbitrary surfaces. In this work, we extend the study to twill weaving, which is used in fabrics such as denim or gabardine. Biaxial twill is a textile weave in which the weft (filling) threads pass over and under two consecutive warp threads and each row is obtained from the row above it by a shift of 1 unit to the right or to the left. The shift operation creates the characteristic diagonal pattern that makes the twill fabric visually appealing.

In this work, we introduce the definition of a twill as a cyclic weaving structure on general surfaces. Based on this definition, we prove that three mesh conditions are necessary and sufficient to obtain twill weaving from a given mesh. We show that many arbitrary meshes do not satisfy these three conditions. It is, therefore, not possible to obtain exact twill for many meshes. On the other hand, for mostly $(4,4)$ meshes, i.e. meshes with large areas of quadrilaterals with 4-valent vertices, it is possible to obtain a reasonably good result of twill in most places. Based on this intuition, we have developed a voting algorithm that guarantees to satisfy most of the twill conditions that allow to demonstrate diagonal patterns everywhere as shown in examples in Figure 1.(a), (b) and (c).

The voting algorithm also creates exact twill if the mesh is twillable. The subdivided meshes are good candidates for twillable meshes since subdivisions can make the number of crossings in each cycle divisible by 4 and they can populate a mesh with regular regions.

Triaxial twill is created from meshes that are populated with $(3,6)$ regions (i.e triangles with 6-valent vertices). Such meshes can be obtained by triangular subdivision schemes, such as midpoint subdivision. We prove that every mesh obtained by a midpoint subdivision is twillable.

Patterns created by triaxial twill are visually interesting and remind some of the M. C. Escher's tilings as shown in Figure 1.(d). However, triaxial twill does not demonstrate the characteristic diagonal pattern of biaxial twill, which can only be obtained from meshes populated by $(4,4)$ regions.

Any mesh can be populated by $(4,4)$ regions by iteratively applying a quad-remeshing scheme such as Catmull-Clark or Doo-Sabin subdivisions. We have investigated the meshes that can be converted to or can continue to be twillable after applications of quad-remeshing schemes. We call these meshes descendent twillable meshes.

We have identified a set of conditions to obtain descendent twillable meshes with quad-remeshing schemes as shown in Figure 2. We prove that threads of such biaxial twill woven objects can always be classified as warp and weft and can be two-colored. We have also developed a coloring algorithm to paint threads of woven object with minimum number of colors. All images in this work are created using this minimum coloring algorithm.

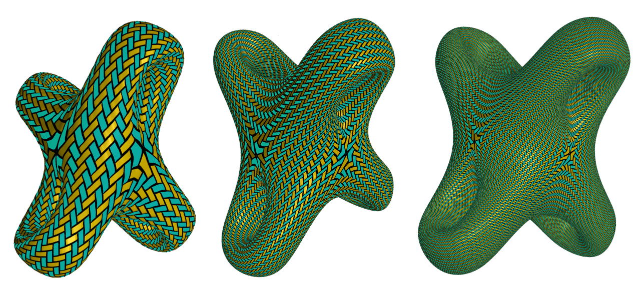


Figure 2: *Three biaxial twill weaving objects obtained by the descendants of the same descendent-twillable mesh.*

References

AKLEMAN, E., CHEN, J., XING, Q., AND GROSS, J. 2009. Cyclic plain-weaving with extended graph rotation systems. *ACM Transactions on Graphics; Proceedings of SIGGRAPH'2009*, 78.1–78.8.

SIGGRAPH 2010, Los Angeles, California, July 25 – 29, 2010.
ISBN 978-1-4503-0210-4/10/0007